BELLCOMM, INC.
1100 Seventeenth Street, N.W. Washington, D.C. 20036

SUBJECT: An Interim Report on an Investigation of a Simplified TLI Hypersurface Representation - Case 310

DATE: March 15, 1967

FROM: B. G. Niedfeldt

ABSTRACT

The object of this memorandum is to investigate the Translunar midcourse correction ΔV penalties associated with the use of a simplified hypersurface representation which could be used during Launch Vehicle guided Translunar Injections on LOR missions. In addition, this memorandum describes the sensitivities of Translunar midcourse corrections to deviations of the IGM guidance parameters and includes a discussion of the effects of correlation between energy and eccentricity. The investigation was performed for three trajectories which span the launch window for February 20, 1968. It was demonstrated that the sensitivities of translunar midcourse corrections to deviations of the IGM guidance parameters were almost identical for all three trajectories.

The first and second midcourse correction requirements were computed using a single set of IGM guidance parameters (an average set) throughout the entire window. Since the variation in guidance parameters over a launch window was found to be very trajectory sensitive, using an average set for the entire 4 1/2 hour window resulted in midcourse corrections that were trajectory sensitive. It was concluded that while some trajectories would allow the use of an average set of guidance parameters, others would require a reduction in launch window in order to keep the midcourse corrections reasonable.

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MEMORANDUM FOR FILE

INTRODUCTION

The object of this memorandum is to investigate the translunar midcourse correction ΔV penalties associated with the use of a simplified hypersurface definition which could be used during Launch Vehicle guided Translunar Injections on LOR missions. The hypersurface, as defined by MSFC, is the collection of all guidance targetting parameters required for use of the Iterative Guidance Mode (IGM) equations to perform Translunar Injections (TLI) across an entire launch opportunity which satisfy the given set of mission constraints. At the present time, MSFC is planning to store, in the on-board computer, the guidance targetting parameters for TLI in table lookup form for two launch opportunities (i.e., first and second Pacific, first Pacific and second Atlantic, etc.).

The digital computer simulation (Reference 1) used in performing this investigation contains a slightly different representation of the hypersurface parameters than is currently being proposed by MSFC. The version used is called Hypersurface I Equations in Reference 2. The guidance parameters used are 1) a unit aim vector which was selected as the vector pointing toward the intersection of a sphere whose radius is equal to the radial distance to the Moon's sphere of influence on the integrated trajectory and the osculating orbit as defined by the nominal TLI state vector, 2) a magnitude which corresponds to the radial distance to the intersection of the unit aim vector and the desired trajectory, 3) a nominal eccentricity, and 4) a term corresponding to twice the conic energy at nominal TLI cutoff. This version of the hypersurface parameters leads to slightly larger midcourse ΔV penalties than the ones currently proposed by MSFC, but the differences are not great.

[redacted]

The simplified hypersurface definitions evaluated in this investigation were determined in the following manner:

1) The latitude and longitude of the unit aim vector in selenographic coordinates were determined across an entire launch opportunity. The average latitude and the average longitude were computed and these values held fixed across the entire opportunity. These two parameters (i.e., the average latitude and longitude) were then used to define the orientation of the average unit aim vector for the entire opportunity. Note that the unit aim vector's orientation is changing across the opportunity at an angular rate equal to the Moon's orbital rate.

2) The average radial distance (throughout the opportunity) to the intersection of the extended unit aim vector and the desired trajectory was computed.

3) The value of the average eccentricity across the entire opportunity was computed.

4) The value of the average energy relate term across the entire opportunity was computed.

This set of average hypersurface parameters were then used as the guidance targetting parameters for use by the IGM equations to perform the TLI maneuvers across the entire launch opportunity. The midcourse corrections required to achieve the desired end conditions were then computed to determine the penalty associated with this simplified hypersurface representation. Special note was made of the effect of correlation between the variations of energy and eccentricity.

The actual investigation was performed using perturbation techniques which are equivalent to the method described above. The technique used in the investigation is fully described in the next section.

GENERAL DISCUSSION OF TECHNIQUES USED

A set of reference trajectories was defined which satisfied all LOR mission objectives and constraints. From the portion of each reference trajectory corresponding to TLI a set of discrete guidance parameters was defined. The reference trajectories used during this investigation were those associated with the launch window of February 20, 1968 (launch azimuths of 72°, 90° and 108°) for the first Pacific opportunity. The TLI maneuvers were made in the plane defined by the Earth Parking Orbit.

Linearized, free fall, transition matrices were generated relating TLI to the first midcourse correction point and the first midcourse correction point to the second midcourse correction point. During the course of the investigation, it was found that second order effects from TLI to the first midcourse correction point were not neglectible and the free fall model for this segment was modified to take these into account. This was accomplished by generating a second order free fall transition matrix between these two points on the trajectory. The mixed or cross partial derivatives, e.g., $\frac{\partial^2 u}{\partial v \partial w}$, were not incorporated into the model.

The first midcourse correction was made 5 hours after TLI and the second midcourse correction made at the Moon's sphere of influence (MSI). The simulation of midcourse corrections was such that the first correction constrained time of arrival at a fixed point at the MSI and the second correction, made at the nominal time of MSI, restored the trajectory to the reference.

The following technique was used to determine the sensitivity matrices. It was assumed that the nominal set of guidance parameters would result in a perfect TLI (i.e., zero midcourse corrections) for the nominal performance ease. The parameters were perturbed one at a time and the deviations from the nominal in position and velocity at a fixed time past nominal cutoff were determined. These deviations were used as input data to determine the midcourse corrections required to get back on to the reference trajectory at MSI. The perturbations studies were the following set:

1. Fractional change in desired eccentricity

2. Fractional change in desired energy term

3. Fractional change in desired aim vector magnitude

4. Rotation of desired unit aim vector about an axis in the desired orbital plane and perpendicular to the nominal unit aim vector.

5. Rotation of desired unit aim vector about an axis out of the desired orbital plane.

Any perturbation of the guidance parameters can be written in terms of these 5 parameters. However energy, eccentricity and perigee magnitude are simply related.

$$e = 1 + \frac{r_p}{\mu} C_3$$

Where e is the eccentricity

C_3 the energy related term

r_p the perigee magnitude

and μ is the gravitational constant

Thus if the perigee magnitude is constant

$$\Delta e = \frac{r_p}{\mu} \Delta C_3$$

and this direct relation makes the first two sensitivities combine into one. Note that although Δe and ΔC_3 are positively correlated, the fractional changes, i.e., $\Delta C_3/C_3$ and $\Delta e/e$, are negatively correlated since, for elliptical orbits, C_3 is negative.

For cases where perigee magnitude is fixed, any perturbation of the guidance parameters can be written in terms of four independent sensitivities: one associated with the energy-eccentricity perturbation and three associated with the aim vector. This is the principal set of deviations which are considered in detail in the next section of this report. The correlation between the energy and eccentricity terms, for the special case of free return Earth-Moon trajectories, has been pointed out in Reference 3 and by a recent MSFC proposal to represent this portion of their hypersurface parameters by a table lookup for C_3 and one value of perigee magnitude per opportunity. Make special note of the fact that if the perigee magnitude is not a constant across the entire launch opportunity, that the assumption will be violated and that this will cost midcourse ΔV. The equation which relates eccentricity deviations to perigee deviations with a constant (or correct) C_3 is as follows:

$$\Delta e = \frac{\Delta r_p}{\mu} C_3$$

The midcourse ΔV penalty for an error in perigee is directly portional to an error in eccentricity by the above equation. For the launch opportunities reviewed to date, the assumption of a constant perigee across an entire opportunity caused a midcourse ΔV penalty of less than 2 feet per second. This amount is considered neglectible.

DISCUSSION OF RESULTS

The midcourse correction sensitivity matrices, which relate individual guidance parameter deviations to midcourse ΔV are given in Table 1 for the three trajectories. The midcourse correction sensitivity matrices which relate energy deviations correlated with a change in eccentricity are given in Table 2. In addition, Figures 1 thru 6 show the magnitude of the first and second midcourse correction ΔV's versus deviations in the guidance parameters.

In both Table 1 and Table 2, the velocity components of the first and second midcourse corrections are given in u v w coordinate systems erected at the nominal first and second midcourse correction points respectively. The u axis is along the nominal radius vector, the v axis is in the desired orbital plane 90° ahead of u, and the w axis is out of the desired orbital plane completing the orthogonal set. The top three rows of each matrix relate first midcourse correction sensitivities to hypersurface discrete deviations and the bottom three rows relate second midcourse correction sensitivities to hypersurface discrete deviations. The hypersurface discrete deviations must be given in terms of radians for the two rotations and in terms of fractional change for the other deviations. The output dimensions are in feet per second.

One feature of the generated miccourse sensitivities (refer to Tables 1 and 2) is that they are relatively constant across the entire launch window. All dominant first and second order terms have the same general magnitude and sign, while only the smaller (and in all cases insignificant) terms vary quite a bit in magnitude but not in sign.

Figures 1 thru 5 show that, for midcourse correction ΔV magnitude requirements of less than 25 feet per second, only energy and eccentricity deviations (independent) lead to noticeable second order effects and then, when the compensating correlation which exists between them is taken into account (i.e., Figure 6), becomes a first order function also. This means that even though there are second order terms contained in the sensitivity matrices (Tables 1, and 2), the higher order terms do not play a dominant role until things are already well out-of-hand.

The effects of taking the correlation between energy and eccentricity into account are most interesting. As will be noted when comparing Figures 3, 4, and 6, the effects of the correlation are very obvious. The magnitude of the midcourse ΔV is markedly reduced and the second order effects were made almost negligible (for the range of deviations shown on the figures).

The best estimate of the range of values over which the deviations of the hypersurface qualities could vary from their true values, summarized from the launch opportunities review to date, is plus and minus the values contained in Table 3. This set of deviations resulted when an average hypersurface set of discretes was established for the first Atlantic opportunity on February 20, 1968, and were by far the largest encountered during the entire month of February 1968 (see Reference 3). The results of multiplying these deviations by the sensitivity matrices given in Tables 1 and 2 are given in Table 4. As can be seen, energy and eccentricity deviations (independent) lead to extreme midcourse requirements ($\approx$ 695. fps), but when the correlation present was taken into account, the midcourse ΔV magnitude dropped markedly ($\approx$ 45. fps). Even with this tremendous reduction in midcourse ΔV, the correlated energy and eccentricity deviation contribution was the largest. The midcourse ΔV requirement for the two rotational errors was also excessive. Deviations selected from other launch opportunities in February of 1968 resulted in the energy and correlated eccentricity terms playing only a minor role in the midcourse correction requirements.

Figures 1, 2, 5, and 6 can be used to estimate the relative midcourse ΔV expense for different launch opportunities. Although the sensitivity matrices may vary slightly from window to window, the conic elements of the family of Earth-Moon free-return trajectories do not vary over an extreme range and therefore, the resultant sensitivity matrices for hypersurface deviations should be relatively constant.

Bounds could be established on the four figures (i.e., 1, 2, 5, and 6) such that the first midcourse correction ΔV should not exceed a given amount and if any deviation for a given opportunity window exceeded that specified amount, the window could be reduced in duration until all the deviations required a midcourse ΔV of less than the specified amount. Then, it could be stated that the reduced window could be represented by a single set of average hypersurface discretes. Note that the duration of the window would have to be reduced such that the midcourse ΔV penalties associated with each opportunity was less than the specified amount.

CONCLUSIONS

It was demonstrated that the sensitivities of translunar midcourse corrections to deviations of the IGM guidance parameters were almost constant over an entire launch opportunity.

It was also demonstrated that there are launch opportunities which exist where a set of average hypersurface discretes (used throughout the entire window) could result in excessive midcourse ΔV expenditure. It was concluded that while some trajectories would allow the use of an average set of guidance parameters, others would require a reduction in launch window in order to keep the midcourse corrections reasonable.

B. G. Niedfeldt

2012-BGN-jan B. G. Niedfeldt

Attachments
Tables 1-4
Figures 1-6

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2. "Apollo Reference Mission Program Iterative Guidance Scheme", TRW Systems, 3838-4003-RU000, NAS 9-4810, December 15, 1965.

3. "Preliminary Results of nn Investigation of the Launch Vehicle Guidance Hypersurface", Memorandum for File, Case 310, D. R. Anselmo, W. D. Kinney, October 6, 1966.

134.48 $-113.06 \times 10^3 \Delta\theta_V$	-2383.3 $-28.89 \times 10^3 \Delta\theta_W$	470.8 $-325.3 \times 10^3 \Delta C_3$	$-3540.$ $-71430. \times 10^4 \Delta e$	-506.7	$\Delta\theta_V$ (Radians)
-94.95 $+82.91 \times 10^3 \Delta\theta_V$	8121. $-9.667 \times 10^3 \Delta\theta_W$	563.6 $-87.9 \times 10^3 \Delta C_3$	$-4625.$ $-17655. \times 10^4 \Delta e$	1728.5	$\Delta\theta_W$ (Radians)
8532.	3.75	6.33	285.5 $+24.4 \times 10^4 \Delta e$	0.0	$\Delta C_3/C_3$ (Fractional change)
-5.972 $+.1794 \times 10^3 \Delta\theta_V$	-475.5 $+9.61 \times 10^3 \Delta C_3$	104.2 $+2008. \times 10^4 \Delta e$	7769.	-101.2	$\Delta e/e$ (Fractional change)
$+316.3$ $-29.38 \times 10^3 \Delta\theta_V$	$-1838.$	220.5 $-3.733 \times 10^3 \Delta C_3$	16565. $-1135. \times 10^4 \Delta e$	-391.2	$\Delta M/M$ (Fractional change)
$-5573.$	-83.13	6.832 $-.3606 \times 10^3 \Delta C_3$	577.9 $-97.9 \times 10^4 \Delta e$	-17.61	

TABLE 1a

TRANSLUNAR MIDCOURSE CORRECTION SENSITIVITY MATRIX FOR DEVIATIONS OF IGM HYPERSURFACE PARAMETERS FEBRUARY 20, 1968, 72° LAUNCH AZIMUTH

130.55 $-113.53 \times 10^3 \Delta\theta_V$	-2387. $-29.63 \times 10^3 \Delta\theta_W$	491.9 $-329.3 \times 10^3 \Delta C_3$	-2266. $-73080. \times 10^4 \Delta e$	-505.8 $-2.228 \times 10^3 \Delta M$	$\Delta\theta_V$ (Radians)
-71.86 $+83.47 \times 10^3 \Delta\theta_V$	8130. $-9.444 \times 10^3 \Delta\theta_W$	539.3 $-84.53 \times 10^3 \Delta C_3$	-4984. $-17900 \times 10^4 \Delta e$	1719.	$\Delta\theta_W$ (Radians)
8471.	1.119	3.292 $+.1007 \times 10^3 \Delta C_3$	169.4 $+24.49 \times 10^4 \Delta e$	0.0	$\Delta C_3/C_3$ (Fractional Change)
-.9757 $+.1749 \times 10^3 \Delta\theta_V$	-477.4	105.3 $+9.608 \times 10^3 \Delta C_3$	7861.2 $+2060 \times 10^4 \Delta e$	-100.9	$\Delta e/e$ (Fractional Change)
61.49 $-29.45 \times 10^3 \Delta\theta_V$	-1840.	224. $-4.566 \times 10^3 \Delta C_3$	16790 $-1210. \times 10^4 \Delta e$	-389.0	$\Delta M/M$ (Fractional Change)
-5547.	-13.45	0.0	41.53 $-17.83 \times 10^4 \Delta e$	-2.666	

TABLE 1b

TRANSLUNAR MIDCOURSE CORRECTION SENSITIVITY MATRIX FOR DEVIATIONS OF IGM HYPERSURFACE PARAMETERS FEBRUARY 20, 1968, 90° LAUNCH AZIMUTH

136.15 $-121.17 \times 10^3 \Delta\theta_V$	-2377.6 $-30.48 \times 10^3 \Delta\theta_W$	466.07 $-339.16 \times 10^3 \Delta C_3$	-3561. $-72437. \times 10^4 \Delta e$	-505.13 $-2.089 \times 10^3 \Delta M$	$\Delta\theta_V$ (Radians)
-96.127 $+87.824 \times 10^3 \Delta\theta_V$	8122.8 $-9.167 \times 10^3 \Delta\theta_W$	558.75 $-78.56 \times 10^3 \Delta C_3$	-4622.5 $-17572 \times 10^4 \Delta e$	1724.4	$\Delta\theta_W$ (Radians)
8547.5	4.26	6.852	291. $+24.3 \times 10^4 \Delta e$	0.0	$\Delta C_3/C_3$ (Fractional Change)
-7.730 $+.179 \times 10^3 \Delta\theta_V$	-474.87	104.45 $+9.52 \times 10^3 \Delta C_3$	7791. $+2024. \times 10^4 \Delta e$	-100.8	$\Delta e/e$ (Fractional Change)
359.89 $-27.683 \times 10^3 \Delta\theta_V$	-1837.	221.43 $-5.803 \times 10^3 \Delta C_3$	16639. $-1189. \times 10^4 \Delta e$	-390.	$\Delta M/M$ (Fractional Change)
-5560.7	-95.49	8.1467 $-.5444 \times 10^3 \Delta C_3$	682.95 $-114.16 \times 10^4 \Delta e$	-20.26	

TABLE 1c

TRANSLUNAR MIDCOURSE CORRECTION SENSITIVITY MATRIX FOR DEVIATIONS OF IGM HYPERSURFACE PARAMETERS FEBRUARY 20, 1968, 108° LAUNCH AZIMUTH

72° Launch Azimuth	90° Launch Azimuth	108° Launch Azimuth
-555.53 $-88.289\ \Delta C_3$	-550.29 $+415.80\ \Delta C_3$	-553.53 $-533.33\ \Delta C_3$
-676.17 $+688.89\ \Delta C_3$	-663.13 $+436.05\ \Delta C_3$	-671.88 $+805.56\ \Delta C_3$
0.0	0.0	0.0
68.572 $-36.111\ \Delta C_3$	67.720 $-33.975\ \Delta C_3$	68.103 $-26.667\ \Delta C_3$
148.09 $-126.11\ \Delta C_3$	144.83 $-57.309\ \Delta C_3$	147.14 $-154.44\ \Delta C_3$
6.0570 $-2.333\ \Delta C_3$	.950 $-.3506\ \Delta C_3$	6.9083 $-12.778\ \Delta C_3$
$\left[\Delta C_3/C_3\ \text{(Fract. Change)}\right]$	$\left[\Delta C_3/C_3\ \text{(Fract. Change)}\right]$	$\left[\Delta C_3/C_3\ \text{(Fract. Change)}\right]$

TABLE 2

TRANSLUNAR MIDCOURSE CORRECTION SENSITIVITIES OF THE ENERGY RELATED TERM CORRELATED WITH THE ECCENTRICITY TERM TO MAINTAIN A CONSTANT PERICENTER

$\Delta\theta_V$	=	.003 radians
$\Delta\theta_W$	=	.002 radians
$\frac{\Delta C_3}{C_3}$	=	.045 fractional change
$\frac{\Delta e}{e}$	=	.00095 fractional change
$\frac{\Delta M}{M}$	=	.0030 fractional change
$\frac{\Delta C_3}{C_3}$	=	.045 fractional change correlated with a $-.00095\ \frac{\Delta e}{e}$ fractional change

TABLE # 3

A SET OF MAXIMUM DEVIATIONS OF HYPERSURFACE QUANTITIES ACROSS AN ENTIRE 4 1/2 HOUR LAUNCH WINDOW

LAUNCH AZIMUTH	ΔV_1 (ft/sec)			ΔV_2 (ft/sec)			DEVIATED PARAMETER
	ΔV_{U_1}	ΔV_{V_1}	ΔV_{W_1}	ΔV_{U_2}	ΔV_{V_2}	ΔV_{W_2}	
72°Az	-.614	.461	25.6	-.016	+.684	-16.7	
90°Az	-.630	.536	25.4	-.001	-.081	-16.6	$\Delta\theta_V$
108°Az	-.682	.502	25.6	-.022	.806	-16.7	(.003 rads)
72°Az	-4.88	16.2	.008	-.951	-3.68	-.166	
90°Az	-4.89	16.2	.002	-.955	-3.68	-.0269	$\Delta\theta_W$
108°Az	-4.88	16.2	.009	-.950	-3.67	-.191	(.002 rads)
72°Az	-638.	-153.	.280	24.2	2.36	-.423	
90°Az	-645.	-147.	.352	24.2	.87	0.0	$\Delta C_3/C_3$
108°Az	-666.	-134.	.308	24.0	-1.79	-.736	(.045 Fract. Change)
72°Az	-648.	-164.	.490	25.5	5.49	-.330	$\Delta e/e$
90°Az	-662.	-166.	.380	26.1	5.03	-.120	
108°Az	-657.	-163.	.500	25.7	5.08	-.380	(.00095 Frac. Change)
72°Az	-1.52	5.19	0.0	-.304	-1.17	-.053	
90°Az	-1.50	5.16	0.0	-.303	-1.17	-.008	$\Delta M/M$
108°Az	-1.53	5.17	0.0	-.302	-1.17	-.061	(.0030 Frac. Change)
72°Az	-25.2	-29.0	0.0	3.01	6.41	.268	
90°Az	-23.9	-29.0	0.0	2.98	6.40	.542	$\frac{C_3}{C_3} = .045$
108°Az	-23.8	-28.6	0.0	3.01	6.31	.285	Correlated with $\frac{e}{e} = -.0009$

TABLE #4

MIDCOURSE CORRECTION REQUIREMENTS FOR A
TYPICAL SET OF HYPERSURFACE PARAMETER DEVIATIONS

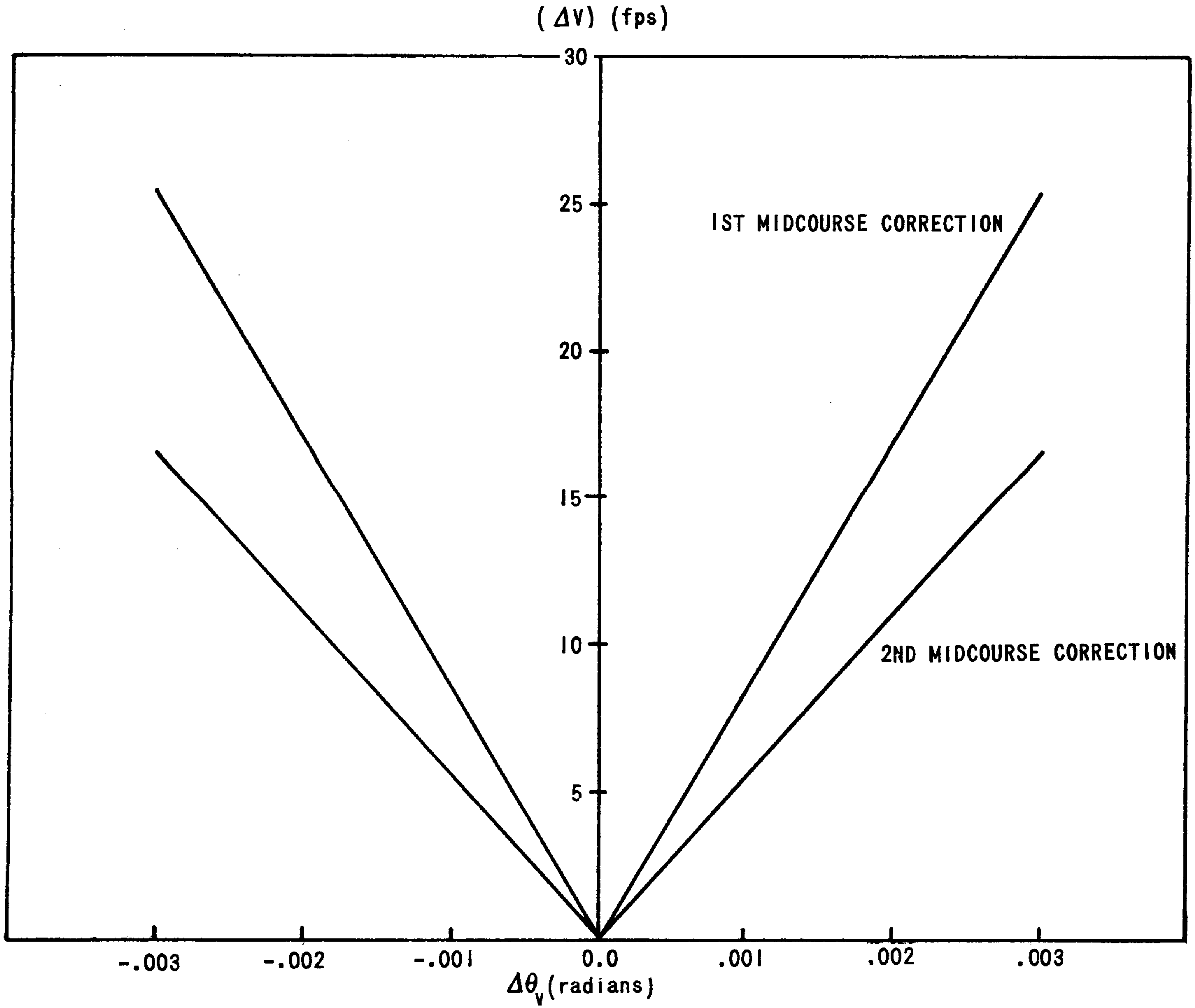

FIGURE 1 - MIDCOURSE CORRECTION ΔV MAGNITUDE AS A FUNCTION OF A ROTATIONAL ERROR (OUT-OF-PLANE) OF THE UNIT AIM VECTOR

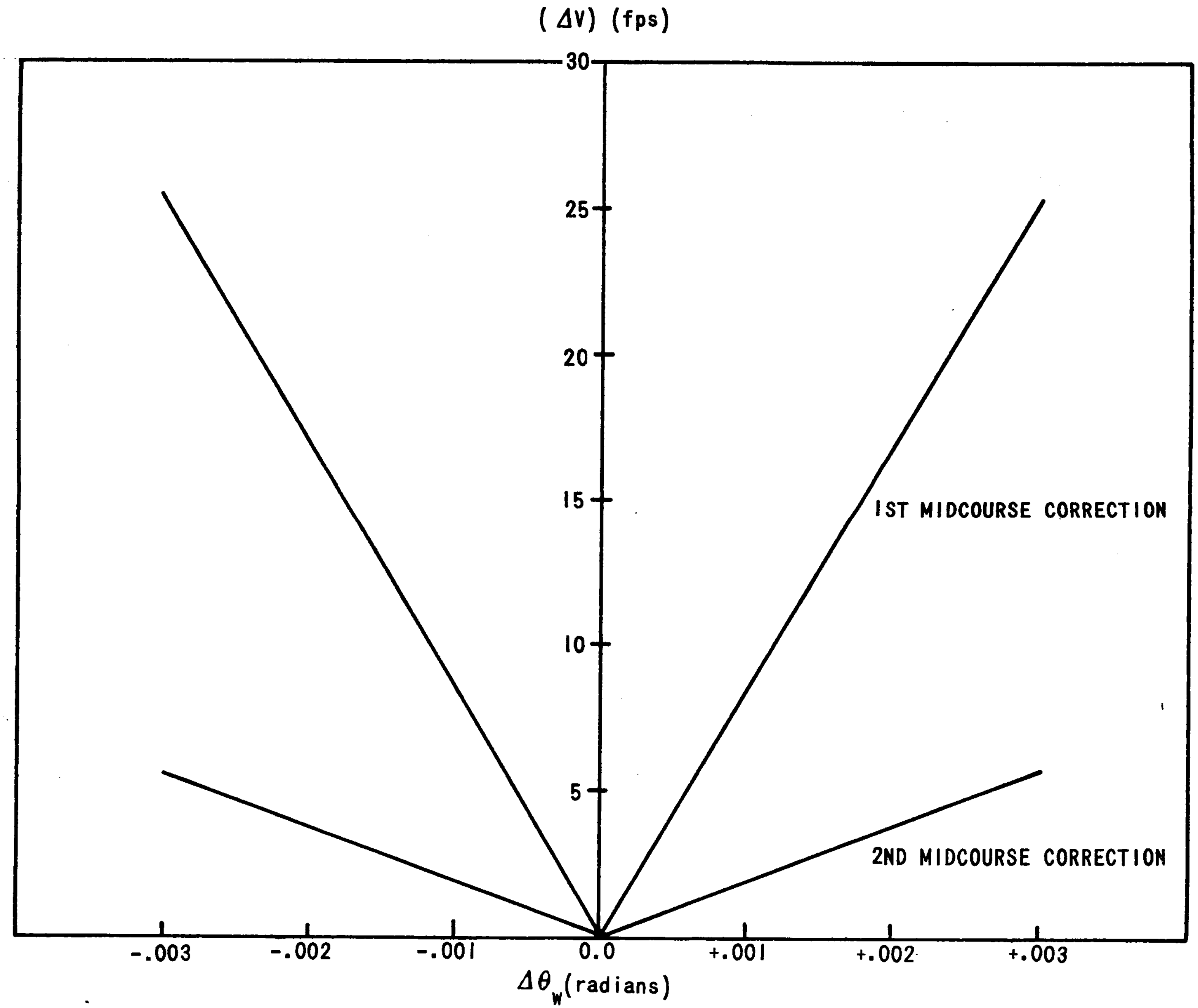

FIGURE 2 - MIDCOURSE CORRECTION ΔV MAGNITUDE AS A FUNCTION OF A ROTATIONAL ERROR (IN-PLANE) OF THE UNIT AIM VECTOR

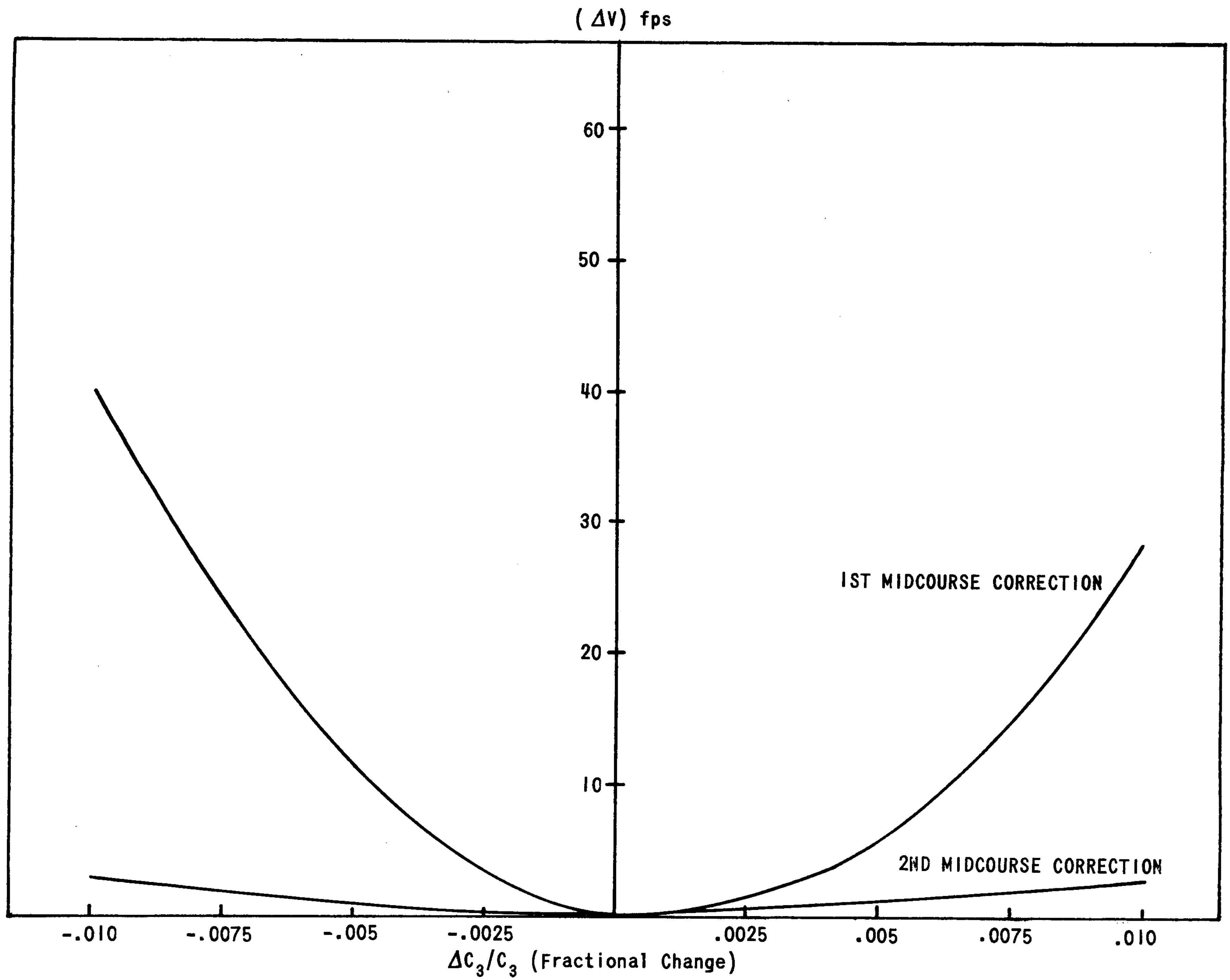

FIGURE 3 - MIDCOURSE CORRECTION ΔV MAGNITUDE AS A FUNCTION OF A FRACTIONAL CHANGE IN THE ENERGY RELATED TERM

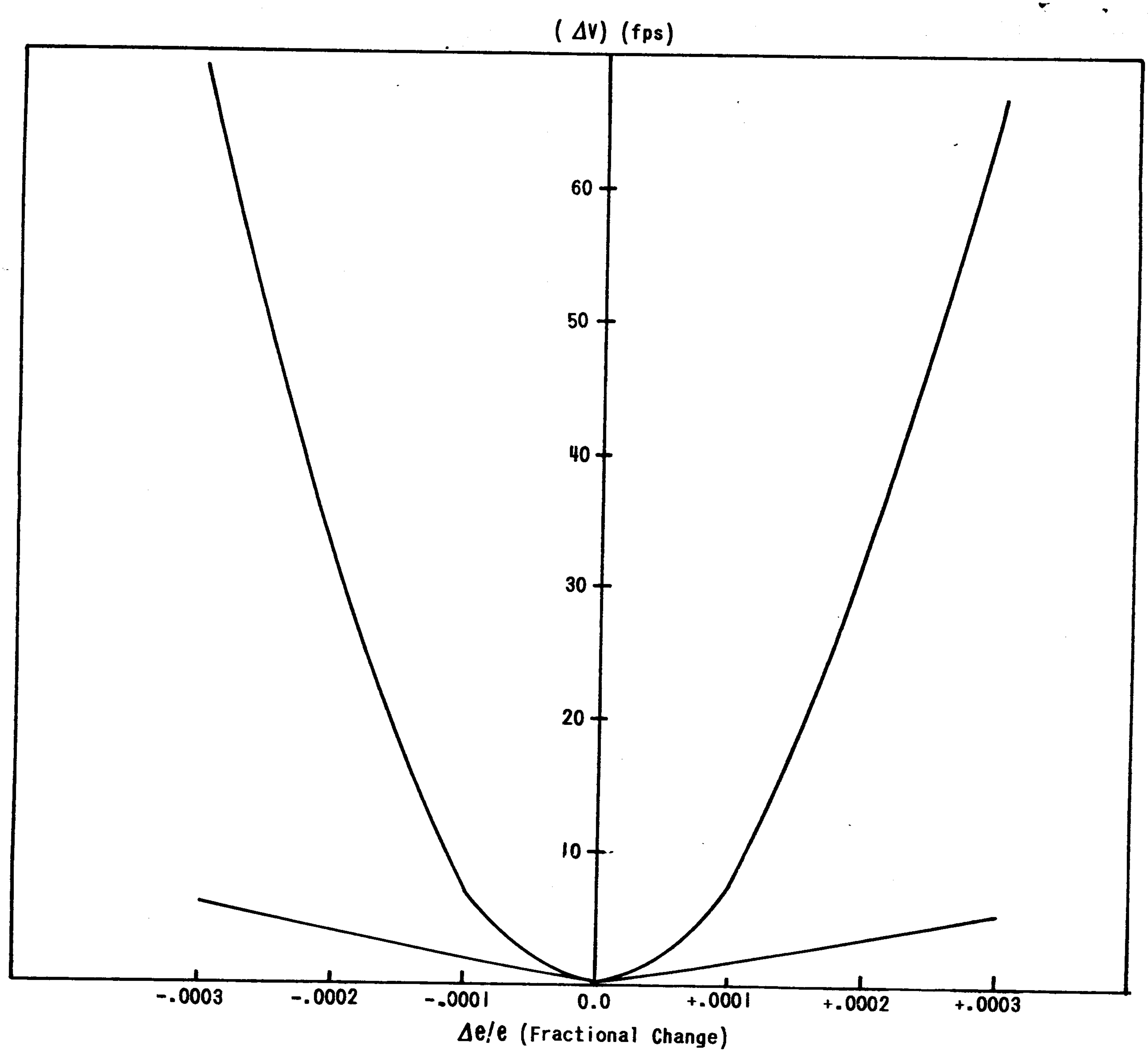

FIGURE 4 - MIDCOURSE CORRECTION ΔV MAGNITUDE AS A FUNCTION OF A FRACTIONAL CHANGE IN ECCENTRICITY

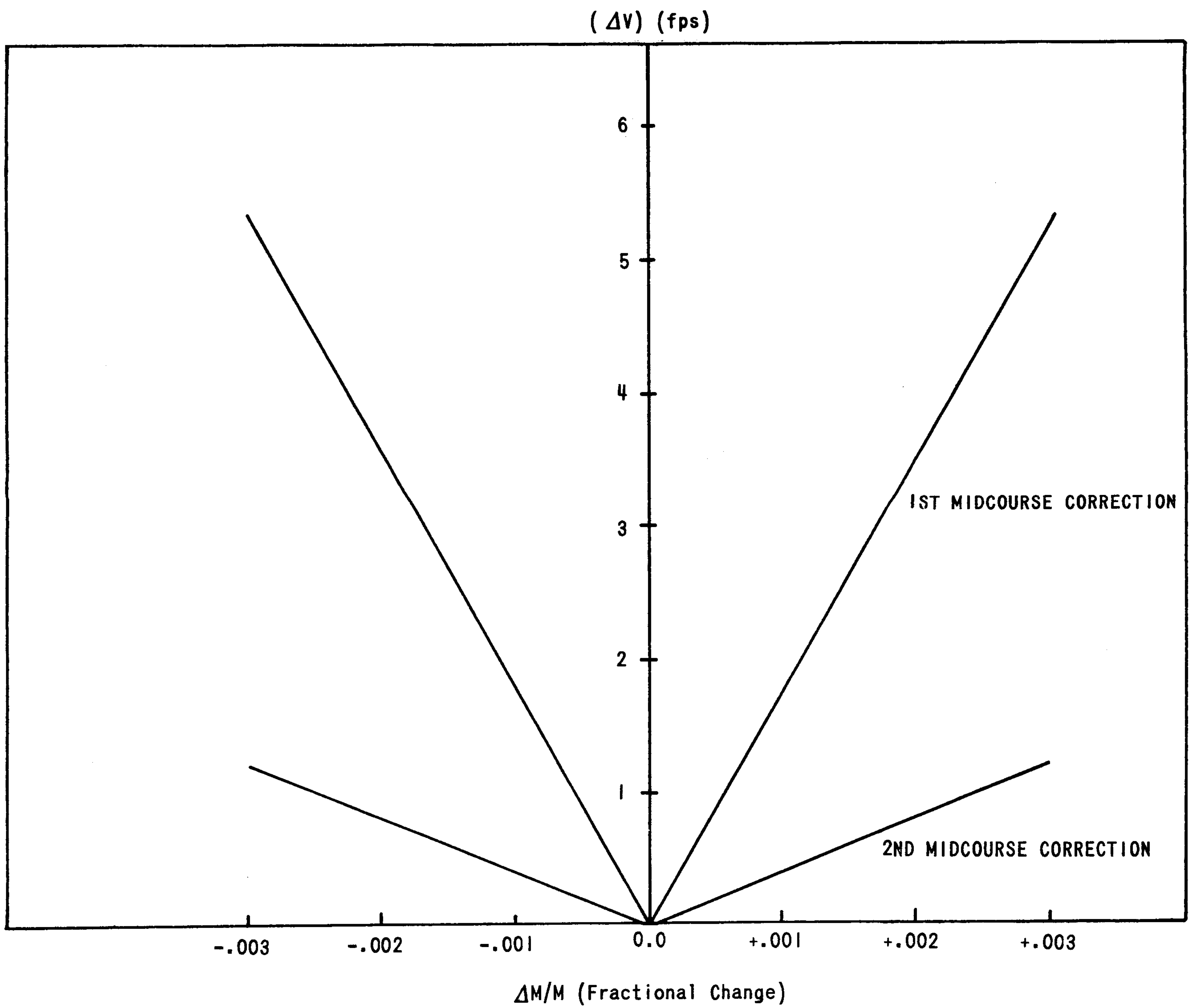

FIGURE 5 - MIDCOURSE CORRECTION ΔV MAGNITUDE AS A FUNCTION OF A FRACTIONAL CHANGE IN THE MAGNITUDE OF THE AIM VECTOR

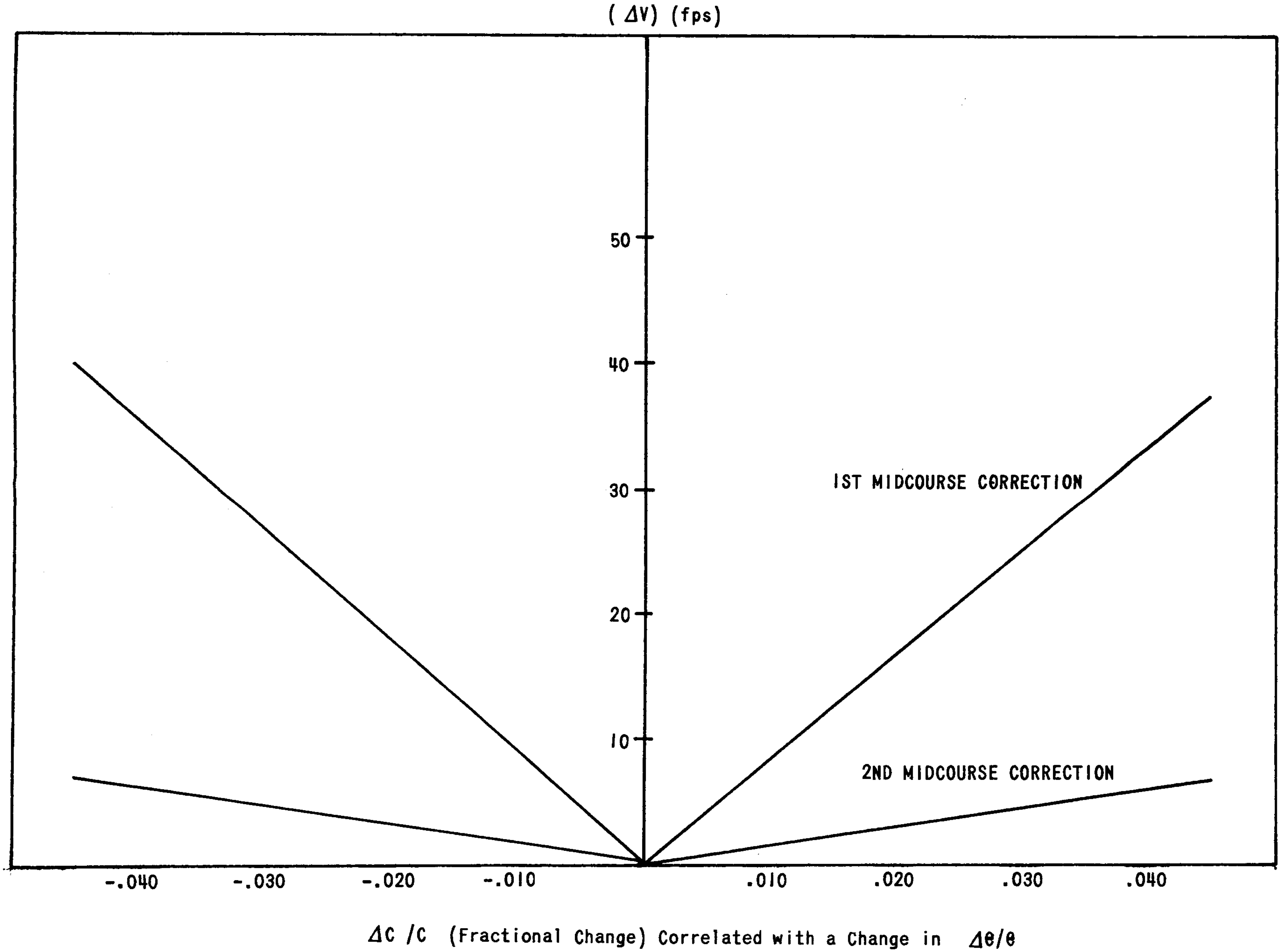

FIGURE 6 - MIDCOURSE CORRECTION ΔV MAGNITUDE AS A FUNCTION OF A FRACTIONAL CHANGE IN THE ENERGY RELATED TERM TAKING INTO ACCOUNT THE CORRELATED ECCENTRICITY CHANGE